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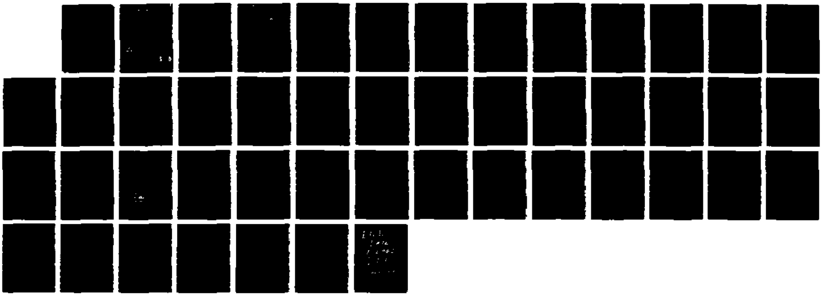

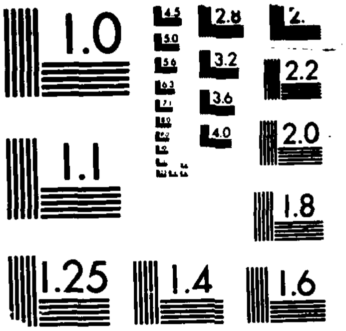
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Contract No. F49620-86-K0009
January 1, 1987 - December 31, 1987

VIBRATION CONTROL OF LARGE STRUCTURES

Submitted to:

Air Force Office of Scientific Research
Building 410
Bolling Air Force Base
Washington, D.C. 20332-6448

Attention: Dr. Anthony K. Amos

Submitted by:

Walter D. Pilkey
Morse Professor

Report Number UVA/525673/MAE88/103

March 1988

SCHOOL OF ENGINEERING AND
APPLIED SCIENCE

UNIVERSITY OF VIRGINIA
CHARLOTTESVILLE, VIRGINIA 22901

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This is a study of some fundamental aspects of the structural dynamics and vibration control of large structures. One focus is the development of a limiting performance formulation with minimum settling time which can accept multiple design objectives efficiently. This new formulation is intended to meet the need of rather comprehensive design objectives for the control of large space structures. Another objective of the study is to develop a systematic way of designing a control system based on the limiting performance characteristics. An indirect synthesis methodology is proposed. It is shown that closed loop control laws can be based on the optimal response trajectories in the time domain. The method is successfully applied to the control of proof-mass actuators. Damping;

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TABLE OF CONTENTS

I. SUMMARY

This is a study of some fundamental aspects of the structural dynamics and vibration control of large structures. One focus is the development of a limiting performance formulation with minimum settling time which can accept multiple design objectives efficiently. This new formulation is intended to meet the need of rather comprehensive design objectives for the control of large space structures. Another objective of the study is to develop a systematic way of designing a control system based on the limiting performance characteristics. An indirect synthesis methodology is proposed. It is shown that closed loop control laws can be based on the optimal response trajectories in the time domain. The method is successfully applied to the control of proof-mass actuators.

II. RESEARCH OBJECTIVES

1. A Comprehensive Limiting Performance Study

A comprehensive limiting performance study was to be performed. In order for the varied and stringent design objectives of large space structure control problems to be met in terms of limiting performance, it was considered essential that a broader limiting performance formulation be developed. In addition to the capability of taking various objectives the resulting trajectories were required to be unique. In conjunction with the synthesis study of feedback control laws, limiting performance characteristics in the modal coordinates were to be provided.

2. Synthesis of Feedback Control Laws

Since the limiting performance characteristics with minimum settling time could give optimal response trajectories in the time domain even when the problems included constraints, it was anticipated that a methodology to give control laws based on these characteristics would be an efficient design tool. An objective of this research was to develop such a methodology. In order to assure robustness and practicality, it was felt that such control laws should be of closed loop form.

3. Inertia Damper Design

The control laws developed were to be applied to the control of proof-mass actuators. Since the control characteristics of proof-mass actuators are dominated by the constraints on the control force and the rattlespace, the control laws should be able to be designed to handle the constraints efficiently.

III. RESEARCH PROGRESS AND STATUS

1. Limiting Performance Characteristics with l_1 Norm and Minimum Settling Time

The limiting performance is defined as the minimum peak value of certain responses while other system responses are constrained. Min-max norms are utilized in the performance index to achieve the limiting performance characteristics. Since the min-max norm employed to minimize the maximum peak value among the peak values of the selected responses does not provide unique peak values except the maximum peak value, an l_1 norm with weights is proposed here to avoid the non-uniqueness of the peak values and to provide flexibility in selecting the performance index. Also, it is shown here how to supplement min-max norms with additional performance measures to settle the responses in minimum time. The formulations presented provide a general approach to find unique optimal response trajectories. The resulting characteristics are referred to as limiting-performance/minimum-settling-time (LP/MST) characteristics.

Problem Statement

A linear vibrating system can be represented by a system of second order linear differential equations

$$M\ddot{\underline{x}} + C\dot{\underline{x}} + K\underline{x} = F\underline{f}(t) \tag{1}$$

where $\underline{x}$ is an n-dimensional vector and M, C, and K are mass, damping, and stiffness matrices with appropriate dimensions, $\underline{f}(t)$ is an nf-dimensional vector representing a forcing function, and F is a coefficient matrix which is used to place the forcing function in the equations of motion. By replacing portions of the system to be designed by generic or control forces which can represent any configuration, the limiting performance characteristics of the system can be found. With the control forces, $\underline{u}(t)$, the system to be studied can be described by the equations of motion

$$M\ddot{\underline{x}} + \bar{C}\dot{\underline{x}} + \bar{K}\underline{x} + V\underline{u}(t) = F\underline{f}(t) \qquad (2)$$

where $\underline{u}(t)$ is an nu-dimensional control force vector, V is a coefficient matrix, and $\bar{C}$ and $\bar{K}$ represent the resulting damping and stiffness matrices after replacing portions of Eq. (1) with control forces. Define a state vector

$$\underline{s} = [\underline{x}^T \ \dot{\underline{x}}^T]^T \qquad (3)$$

Then, Eq. (2) can be represented by the state equations

$$\dot{\underline{s}}(t) = A\underline{s}(t) + B\underline{u}(t) + C\underline{f}(t) \qquad (4)$$

where $\underline{s}(t)$ is a 2n-dimensional state vector, A, B, and C are constant coefficient matrices represented by

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}\bar{K} & -M^{-1}\bar{C} \end{bmatrix} \tag{5}$$

$$B = \begin{bmatrix} 0 \\ -M^{-1}V \end{bmatrix} \tag{6}$$

$$C = \begin{bmatrix} 0 \\ M^{-1}F \end{bmatrix} \tag{7}$$

where I is the identity matrix and 0 is the null matrix.

The problem is to find an optimal control $\underline{u}^*(t)$ based on a performance index as the weighted sum of p system responses

$$\text{minimize} \quad J = \underline{w}^T \underline{j} = w_1 J_1 + w_2 J_2 + \dots + w_p J_p \tag{8}$$

where

$$\underline{j} = [\ J_1\ J_2\ \dots\ J_p\ ]^T = \max_{t_o \le t \le t_f} |P_1 \underline{s} + P_2 \underline{u} + P_3 \underline{f}| \tag{9}$$

$$\underline{w} = [\ w_1\ w_2\ \dots\ w_p\ ]^T \tag{10}$$

where t_o and t_f are given initial and final times, P_1, P_2 and P_3 are prescribed constant coefficient matrices, and $\underline{w}$ is a prescribed constant coefficient vector containing weights. Equations (8) and (9) represent l_1 and min-max norms, respectively. The solution is to be computed such that the peak values of $\underline{j}$ are unique and the response trajectories after the peak values of responses are settled in minimum time.

Constraints are imposed on the dynamic system under study. The

format of the constraints is

$$\underline{y}_L \leq Q_1\underline{s} + Q_2\underline{u} + Q_3\underline{f} \leq \underline{y}_U \quad \text{for } t_o \leq t \leq t_f \tag{11}$$

where $\underline{y}_L$ and $\underline{y}_U$ are nc-dimensional lower and upper constraint vectors and Q_1, Q_2 and Q_3 are constant coefficient matrices. The quantity nc is the number of constraints

Linear Programming Formulation

Since the problem defined above has a linear performance index and linear constraints, standard linear programming can be used to solve the optimization problem, provided that the problem has not been stated to be physically over-constrained. In this section, a linear programming formulation without imposing minimum settling time is studied. If the system in Eq. (4) is discretized using N uniform time intervals, a set of state difference equations is obtained for k = 1,2,...,N

$$\underline{s}(k) = G\underline{s}(k-1) + H[B\underline{u}(k) + C\underline{f}(k)] \tag{12}$$

where

$\underline{s}(k)$ = state vector at time $t_k = kh$

$\underline{u}(k)$, $\underline{f}(k)$ = control and external excitation vectors, assumed to be constant over the interval $t_{k-1} < t \leq t_k$

$G = e^{Ah}$

$H = \int_o^h e^{A(h-\tau)} d\tau$

h = time interval = $t_k - t_{k-1}$

The state vector, at any time $t = t_k$, can be expressed as a function of the initial state $\underline{s}(0)$, the control history $\underline{u}(1)$, $\underline{u}(2)$, ..., $\underline{u}(N)$, and the external excitation $\underline{f}(1)$, $\underline{f}(2)$, ..., $\underline{f}(N)$. For k = 1,2, ... ,N

$$\underline{s}(k) = G^k \underline{s}(0) + \sum_{j=1}^{k} G^{k-j} H[B\underline{u}(j) + C\underline{f}(j)] \tag{13}$$

The constraints in Eq. (11) are discretized similarly

$$\underline{y}_L(k) \leq Q_1\underline{s}(k) + Q_2\underline{u}(k) + Q_3\underline{f}(k) \leq \underline{y}_U(k) \quad \text{for } k = 1,2,\ldots,N \tag{14}$$

Having discretized the state equations, these constraints can only be enforced at the end of each interval. At "intermediate" times, they may be violated.

Consider now the linear programming formulation of the limiting performance problem. The objective function of Eq. (9), which reflects the min-max norm, can be converted into a constraint set. Since $\underline{j}$ is the vector of maximum values of $|P_1\underline{s}(k) + P_2\underline{u}(k) + P_3\underline{f}(k)|$ for all k,

$$|P_1\underline{s}(k) + P_2\underline{u}(k) + P_3\underline{f}(k)| \leq \underline{j} \tag{15}$$

To place the l_1 norm of the performance index in Eq. (8) into linear programming form, define a (p+nc) x p augmented matrix $\bar{J}$ which has elements of vector $\underline{j}$ along its diagonal as the only non-zero elements, i.e.,

$$
\bar{J} = \left[\begin{array}{ccccc}
J_1 & 0 & . & . & 0 \\
0 & J_2 & 0 & . & 0 \\
. & & . & & . \\
. & & & . & 0 \\
0 & . & . & 0 & J_p \\
\hline
0 & . & . & . & 0 \\
. & . & . & . & . \\
. & . & . & . & . \\
0 & . & . & . & 0
\end{array}\right] \tag{16}
$$

Also, to merge constraints in Eqs. (14) and (15), define augmented matrices

$$
\bar{Q}_1 = \begin{bmatrix} P_1 \\ Q_1 \end{bmatrix}, \quad \bar{Q}_2 = \begin{bmatrix} P_2 \\ Q_2 \end{bmatrix}, \text{ and } \bar{Q}_3 = \begin{bmatrix} P_3 \\ Q_3 \end{bmatrix} \tag{17}
$$

and augmented vectors

$$
\bar{\underline{y}}_U(k) = [\underline{0}^T \ \underline{y}_U^T(k)]^T \text{ and } \bar{\underline{y}}_L(k) = [\underline{0}^T \ \underline{y}_L^T(k)]^T \tag{18}
$$

where $\underline{0}$ is a p-dimensional null vector. Then Eqs. (14) and (15) can be merged as

$$
-\bar{J} + \bar{Q}_1\underline{s}(k) + \bar{Q}_2\underline{u}(k) + \bar{Q}_3\underline{f}(k) \leq \bar{\underline{y}}_U(k) \tag{19}
$$

$$
-\bar{J} - \bar{Q}_1\underline{s}(k) - \bar{Q}_2\underline{u}(k) - \bar{Q}_3\underline{f}(k) \leq -\bar{\underline{y}}_L(k) \tag{20}
$$

Substituting Eq. (13) into Eqs. (19) and (20), the constraint equations have the control sequence $\underline{u}(k)$ as the only unknowns. For $k = 1,2,\ldots,N$

$$-\bar{J} + \bar{Q}_1 \sum_{j=1}^{k} G^{k-j} HB\underline{u}(j) + \bar{Q}_2\underline{u}(k) \leq$$

$$\underline{\bar{y}}_U(k) - \bar{Q}_1 G^k \underline{s}(0) - \bar{Q}_1 \sum_{j=1}^{k} G^{k-j} HC\underline{f}(j) - \bar{Q}_3\underline{f}(k) \tag{21}$$

$$-\bar{J} - \bar{Q}_1 \sum_{j=1}^{k} G^{k-j} HB\underline{u}(j) - \bar{Q}_2\underline{u}(k) \leq$$

$$-\underline{\bar{y}}_L(k) + \bar{Q}_1 G^k \underline{s}(0) + \bar{Q}_1 \sum_{j=1}^{k} G^{k-j} HC\underline{f}(j) + \bar{Q}_3\underline{f}(k) \tag{22}$$

Since the control $\underline{u}(k)$ should be unrestricted in sign, a standard linear programming technique for dealing with such variables is introduced. Define

$$\underline{u}(k) = \underline{U}^{+}(k) - \underline{U}^{-}(k) \tag{23}$$

where

$$\underline{U}^{+}(k) \geq 0 \quad \text{and} \quad \underline{U}^{-}(k) \geq 0 \tag{24}$$

To place this optimization problem into a standard linear programming form, define

$$\underline{z} = \begin{bmatrix} \underline{J} \\ \underline{\underline{u}} \end{bmatrix} \tag{25}$$

where

$$\underline{\underline{u}} = [\underline{U}^{+}(1)^T \ \underline{U}^{-}(1)^T \ \underline{U}^{+}(2)^T \ \underline{U}^{-}(2)^T \ \ldots \ \underline{U}^{+}(N)^T \ \underline{U}^{-}(N)^T]^T \tag{26}$$

and

$$\underline{c}^T = [\ \underline{w}^T \ \underline{0}^T\ ] \tag{27}$$

where $\underline{0}$ is a 2N-dimensional null vector. The vector $\underline{w}$ can be selected to provide unique values of peaks, i.e., J_1, J_2, ..., J_p. Then the linear programming problem is to minimize

$$J = \underline{c}^T \underline{z} \tag{28}$$

subject to the constraints

$$H\underline{z} \leq \underline{b} \tag{29}$$

where H and $\underline{b}$ are a 2N*(p+nc) x (2N*nu+p) coefficient matrix and a 2N*(p+nc) coefficient vector, respectively, representing constraints of Eqs. (21) and (22).

Limiting-Performance/Minimum-Settling-Time Formulation

The unique peak values can be obtained based on the linear programming formulation of the previous section. Formulations are studied to achieve the minimum settling time characteristics after the peak values. Two methods are available to achieve the minimum settling time and they can produce identical minimum settling time responses. One utilizes additional performance indices and the other utilizes additional constraints. The additional performance indices and constraints can be employed independently or simultaneously depending on the requirements of a problem. However, in order to minimize the size of the linear programming problem, additional performance indices for the responses selected in the performance index and additional

constraints for the constrained variables are suggested to be employed.

To provide the minimum settling time characteristics for the responses in the performance index, the performance index of Eq. (8) is separated into two performance indices. Assume that the peak values of the limiting performance occur during the time between t_o and t_t. One of the performance indices, referred to as the transient performance index, is given by

$$J^t = \underline{w}^T \underline{j}^t \tag{30}$$

where

$$\underline{j}^t = \max_{t_o \leq t < t_t} |P_1\underline{s} + P_2\underline{u} + P_3\underline{f}| \tag{31}$$

where t_t is the time limit for the transient period. The other, referred to as the steady-state performance index, is defined as

$$J^s = \underline{w}^T \underline{j}^s \tag{32}$$

where

$$\underline{j}^s = \max_{t_t \leq t \leq t_f} |P_1\underline{s} + P_2\underline{u} + P_3\underline{f}| \tag{33}$$

Note that the same $\underline{w}^T$ and P_1, P_2, P_3 are used in the performance indices. The transient performance index is used to give unique response trajectories up to the point of the peak values and the steady-state performance index is used to settle the response

trajectories after the peaks within the desired values in minimum time. See Fig. 1.

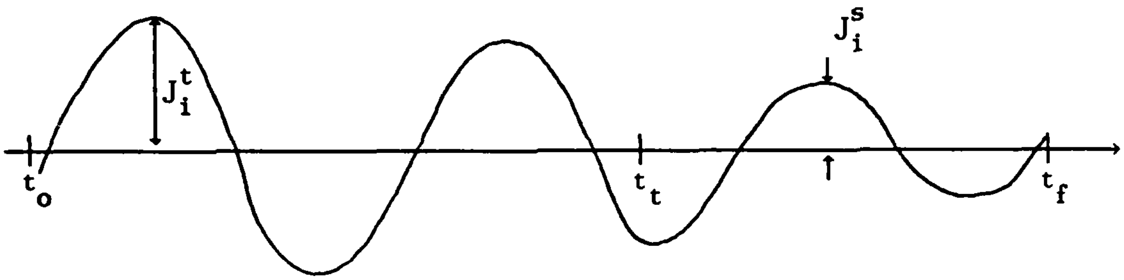

Fig. 1 A Response Trajectory with Transient and Steady-State Performance Indices ($i = 1,2,\ldots,p$)

Now, the global performance index is defined by

$$J^g = J^t + J^s \tag{34}$$

The objective functions of Eqs. (31) and (33), which reflect min-max norms, are also entered into the constraints. Since $\underline{j}^t$ is the vector of the maximum values of $|P_1\underline{s} + P_2\underline{u} + P_3\underline{f}|$ for $t_o \le t < t_t$ and so is $\underline{j}^s$ for $t_t \le t \le t_f$, the constraints from the objective functions are, if they are discretized using the same discretization of Eq. (12),

$$|P_1\underline{s}(k) + P_2\underline{u}(k) + P_3\underline{f}(k)| \le \underline{j}^t \quad \text{for} \quad k = 0,1,\ldots,N^{t-1}$$

and (35)

$$|P_1\underline{s}(k) + P_2\underline{u}(k) + P_3\underline{f}(k)| \le \underline{j}^s \quad \text{for} \quad k = N^t, N^t+1,\ldots,N$$

where N^t is the discretized time limit for the transient period.

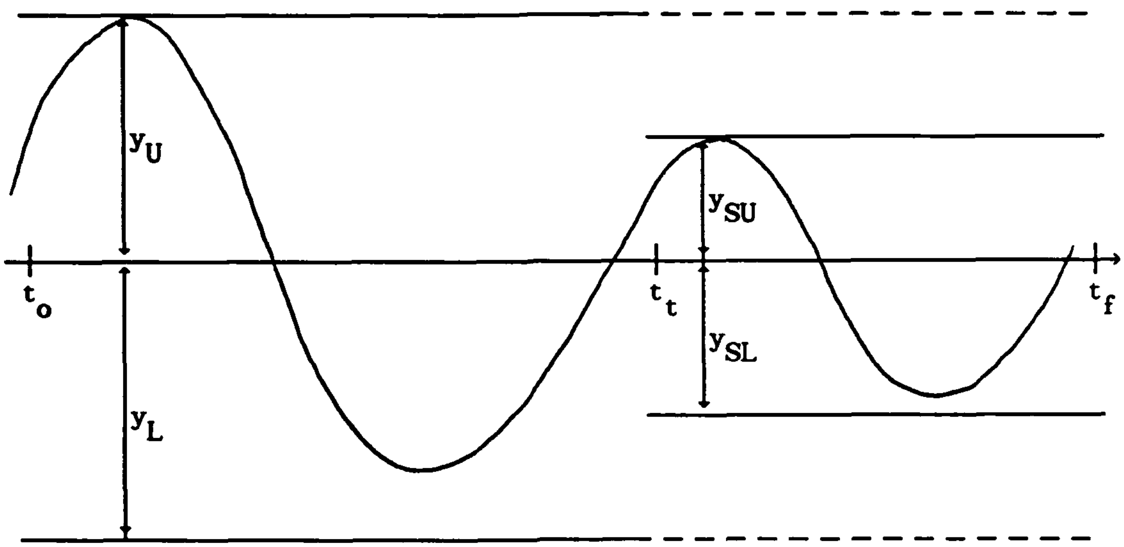

Fig. 2 A Response Trajectory with Transient and Steady-State Constraints

To achieve minimum settling time characteristics for the responses in the constraints, consider two sets of constraints. One set of them, referred to as the transient constraints, is used to represent the original constraints in Eq. (11) from t_o to t_t. An additional constraint set, referred to as steady-state constraints, is imposed from t_t to t_f

$$\underline{y}_{SL} \leq Q_1\underline{s} + Q_2\underline{u} + Q_3\underline{f} \leq \underline{y}_{SU} \quad \text{for } t_t \leq t \leq t_f \tag{36}$$

where $\underline{y}_{SL}$, $\underline{y}_{SU}$ are nc-dimensional coefficient vectors representing lower and upper bounds of steady-state constraints. The steady-state constraints represent the desired values of the responses after the peaks. The responses are required to be settled within these values in

the smallest value of t_t. Note that the matrices Q_1, Q_2, and Q_3 in Eq. (36) are identical with those in Eq. (11). See Fig. 2. for an illustration.

To take the new performance index of Eq. (34) into linear programming, define p x p matrices $\bar{J}^t$ and $\bar{J}^s$ which have elements of vectors $\underline{j}^t$ and $\underline{j}^s$, respectively, along their diagonals as only non-zero elements. Also, define (p+nc) x 2p matrices $\hat{J}^t$ and $\hat{J}^s$ such as

$$\hat{J}^t = \begin{bmatrix} \bar{J}^t & 0 \\ 0 & 0 \end{bmatrix} \tag{37}$$

$$\hat{J}^s = \begin{bmatrix} 0 & \bar{J}^s \\ 0 & 0 \end{bmatrix} \tag{38}$$

where 0 represent null matrices. The two sets of constraints are discretized and merged into the constaints from the performance index, Eq. (35), using augmented matrices and vectors of Eqs. (17) and (18). Then, for $k = 1,2,\ldots,N^{t-1}$, the constraints are

$$-\hat{J}^t + \bar{Q}_1\underline{s}(k) + \bar{Q}_2\underline{u}(k) + \bar{Q}_3\underline{f}(k) \le \bar{\underline{y}}_U(k) \tag{39}$$

$$-\hat{J}^t - \bar{Q}_1\underline{s}(k) - \bar{Q}_2\underline{u}(k) - \bar{Q}_3\underline{f}(k) \le -\bar{\underline{y}}_L(k) \tag{40}$$

and, for $k = N^t, N^t+1,\ldots,N$, the constraints are

$$-\hat{J}^s + \bar{Q}_1\underline{s}(k) + \bar{Q}_2\underline{u}(k) + \bar{Q}_3\underline{f}(k) \le \bar{\underline{y}}_{SU}(k) \tag{41}$$

$$-\hat{J}^s - \bar{Q}_1\underline{s}(k) - \bar{Q}_2\underline{u}(k) - \bar{Q}_3\underline{f}(k) \le -\bar{\underline{y}}_{SL}(k) \tag{42}$$

Substituting Eq. (13) into Eq. (39) to Eq. (42), the constraint equations have control sequence $\underline{u}(k)$ as only unknowns.

To place this optimization problem into a standard linear programming form, define

$$\hat{\underline{z}} = \begin{bmatrix} \underline{j}^t \\ \underline{j}^s \\ \underline{\underline{u}} \end{bmatrix} \tag{43}$$

where $\underline{\underline{u}}$ is given by Eq. (26) and

$$\hat{\underline{c}}^T = [\ \underline{w}^T\ \underline{w}^T\ \underline{0}^T] \tag{44}$$

where $\underline{0}$ is a 2N-dimensional null vector. Then the linear programming problem becomes: minimize

$$J^g = \hat{\underline{c}}^T \hat{\underline{z}} \tag{45}$$

subject to the constraints

$$\hat{H}\hat{\underline{z}} \leq \hat{\underline{b}} \tag{46}$$

where $\hat{H}$ and $\hat{\underline{b}}$ are a 2N*(p+nc) x (2N*nu+2p) coefficient matrix and a 2N*(p+nc) coefficient vector, respectively, representing constraints of Eq. (39) to Eq. (42).

2. Limiting Performance Based Feedback Control Synthesis and Its Application to the Control of Proof-Mass Actuators

The control characteristics of proof-mass actuators are dominated by the constraints on the control force and the rattlespace. This constrained control problem raises a serious difficulty in designing a system using conventional control system design methods such as linear control or optimal control. It has been known that the limiting performance formulation with the minimum settling time can provide an optimal open loop control law for the proof-mass actuators. In order to enhance the practicality and robustness of the open loop control and to provide a systematic approach to handle constrained control problems, feedback control synthesis methods based on the limiting performance trajectories are developed. Some control laws developed with these new approaches for the control of proof-mass actuators are presented.

Suboptimal Design Synthesis of Feedback Control Systems

Methods of designing suboptimal feedback control laws based on the LP/MST characteristics are presented here. Assume that the system is to be controlled by active actuators and the actuators are subject to constraints. The active controllers are assumed to be controlled by a constant state variable feedback and the feedback gain matrix is found based on the LP/MST characteristics. Two distinct design synthesis methods to construct feedback control laws are studied. For general equations of motion including systems with a non-proportional damping matrix, a suboptimal feedback gain matrix is found using the constrained

curve fitting technique based on approximating optimal response trajectories. For decoupled equations of motion, e.g., systems with a proportional damping matrix, another synthesis method can also be utilized. In this method, closed loop eigenvalues are first extracted using the curve fitting based on approximating optimal response trajectories and the eigenvalues are assigned using an eigenvalue assignment method based on a constant state variable feedback to find the feedback gain matrix.

Identification of Feedback Control Based on the Optimal Response Trajectories

One of the two methods to construct feedback control laws based on the LP/MST characteristics is studied here. With this method, the constant state variable feedback gains are obtained directly by fitting the optimal response trajectories. This method can handle equations of motion with a non-proportional damping matrix. However, there is no guarantee that the resulting system will be stable.

Computation of LP/MST Characteristics

The first step is to compute the LP/MST characteristics. By replacing portions of a system to be designed by generic forces which can represent any configuration, the LP/MST characteristics of the system can be found as described previously.

Identification of Constant Feedback Gain Matrix

The optimal LP/MST trajectories $\underline{s}^*(k) = [\underline{x}^*(k)^T \; \dot{\underline{x}}^*(k)^T]^T$ and $\underline{u}^*(k)$

$(k = 1,2,\ldots,N)$ are used as a starting point. Assume that the active controllers are controlled by a constant state variable feedback, i.e.,

$$\underline{u}(k) = D\underline{s}(k) \tag{47}$$

where D is an nu x 2n constant feedback gain matrix. Since nu controllers are considered and optimal control forces are available for each controller, it is possible to proceed controller by controller. For controller j (j = 1,2,...,nu), a suboptimal linear control law to be determined is described by

$$u_j(k) = \begin{cases} u_{max_j} & \text{for } u_j^s(k) > u_{max_j} \\ u_j^s(k) & \text{for } u_{min_j} \leq u_j^s(k) \leq u_{max_j} \\ u_{min_j} & \text{for } u_j^s(k) < u_{min_j} \end{cases} \tag{48}$$

where

$$u_j^s(k) = \underline{d}_j\underline{s}^*(k) \tag{49}$$

$\underline{d}_j = j^{th}$ row of D matrix

Note that Eq. (48) serves to enforce a constraint on the control force of controller j

$$u_{min_j} \leq u_j(k) \leq u_{max_j} \tag{50}$$

Then the problem becomes to find the optimal feedback gains $\underline{d}_j$ (for j = 1,2,...,nu) which minimize

$$H_j = \sum_{k=1}^{N} \Delta_j(k)^2 \tag{51}$$

where

$$\Delta_j(k) = |u_j^*(k) - u_j^s(k)| \tag{52}$$

To find $\underline{d}_j$ efficiently a curve fitting method based on a least squares residual function of Eq. (51) can be used.

Identification of Feedback Control Based on the Assignment of the Optimal Closed Loop Eigenvalues

In this section it is assumed that the equations of motion can be decoupled, i.e., the damping matrix is proportional or the system is undamped. In this method, the optimal response trajectories are calculated in the modal coordinates and the curve fitting technique is used to find closed loop eigenvalues for each mode. Then, the state variable feedback gains are obtained using an eigenvalue assignment technique. This method can guarantee the stability of the resulting gains. Also, the constraints on the response variables can be enforced during the design process.

Computation of LP/MST Characteristics in Modal Coordinates

Replace portions of a system to be designed by generic forces, decouple the equations of motion by using the normal mode method, and compute the limiting performance characteristics in the modal coordinates.

Start from Eq. (2). Using the normal mode method and the transformation

$$\underline{x}(t) = \hat{\Phi}\underline{q}(t) = \sum_{r=1}^{n} \hat{\underline{\phi}}_r q_r(t) \tag{53}$$

where $\hat{\underline{\phi}}_r$ is obtained by solving

$$(\bar{K} - \hat{\omega}_r^2 M)\hat{\underline{\phi}}_r = \underline{0}, \quad r = 1,2,\ldots,n \tag{54}$$

Eq. (2) appears as

$$\hat{M}\ddot{\underline{q}} + \hat{C}\dot{\underline{q}} + \hat{K}\underline{q} + \hat{V}\underline{u} = \hat{F}\underline{f}(t) \tag{55}$$

where

$$\begin{aligned}
\hat{M} &= \hat{\Phi}^T M \hat{\Phi} = \text{modal mass matrix} \\
\hat{C} &= \hat{\Phi}^T \bar{C} \hat{\Phi} = \text{modal damping matrix} \\
\hat{K} &= \hat{\Phi}^T \bar{K} \hat{\Phi} = \text{modal stiffness matrix} \\
\hat{F} &= \hat{\Phi}^T F \\
\hat{V} &= \hat{\Phi}^T V
\end{aligned} \tag{56}$$

By using the state vector

$$\hat{\underline{s}} = [\underline{q}^T \; \dot{\underline{q}}^T]^T \tag{57}$$

Eq. (55) can be represented by a state equation

$$\dot{\hat{\underline{s}}}(t) = \hat{A}\hat{\underline{s}}(t) + \hat{B}\underline{u}(t) + \hat{C}\underline{f}(t) \tag{58}$$

where

$$\hat{A} = \begin{bmatrix} 0 & I \\ -\hat{M}^{-1}\hat{K} & -\hat{M}^{-1}\hat{C} \end{bmatrix} \tag{59}$$

$$\hat{B} = \begin{bmatrix} 0 \\ -\hat{M}^{-1}\hat{V} \end{bmatrix} \tag{60}$$

$$\hat{C} = \begin{bmatrix} 0 \\ -\hat{M}^{-1}\hat{F} \end{bmatrix} \tag{61}$$

The performance index and the constraints can also be transformed with the relationship

$$\underline{s}(t) = \hat{\Phi}\hat{\underline{s}}(t) \tag{62}$$

Then the problem can be solved in the modal coordinates using one of the formulations given previously.

Identification of Suboptimal Closed Loop Eigenvalues

The optimal limiting performance trajectory for each mode in modal coordinates, $q_r^*(k)$ $(r = 1,2,\ldots,n;\ k = 1,2,\ldots,N)$, is used as a starting point. The first task is to find optimal ζ_r^* and ω_r^* for each mode which will minimize

$$H_r' = \sum_{k=1}^{N} \Delta_r'(k)^2 \tag{63}$$

where the measure of approximation is

$$\Delta_r'(k) = |q_r^*(k) - q_r'(k)| \tag{64}$$

where

$$q_r'(k) = q_r'\{\zeta_r, \omega_r\}$$

= The analytical solutions for the time response of the decoupled SDOF system.

To find optimal ζ_r^* and ω_r^*, a least square curve fitting technique is employed. Now the closed loop eigenvalues for each mode can be calculated by the characteristic equation

$$\lambda_r^{*2} + 2\zeta_r^*\omega_r^*\lambda_r^* + \omega_r^{*2} = 0, \quad r = 1,2,\ldots,n \tag{65}$$

or

$$\lambda_r^* = -\zeta_r^*\omega_r^* \pm i\,\omega_r^*\sqrt{1 - \zeta_r^{*2}} \tag{66}$$

Since the closed loop eigenvalues are now known for each mode, eigenvalue assignment techniques can be used to find the feedback gain matrix.

Assignment of Closed Loop Eigenvalues

Several assignment techniques are available, often differing according to the characteristics of eigenvalues to be assigned. From Eq. (4) the system to be designed is written as

$$\dot{\underline{s}}(t) = A\underline{s}(t) + B\underline{u}(t) + C\underline{f}(t) \tag{67}$$

and the problem is to find the feedback gain matrix D such that

$$\underline{u}(t) = D\underline{s}(t) \tag{68}$$

which will produce the specified closed loop eigenvalues λ_r^* (r = 1,2,... ,n) of Eq. (66) from the matrix

$$\bar{A} = A + BD \tag{69}$$

Application to the Control of Proof-Mass Actuators

As a proof-of-the-concept example, the suboptimal design synthesis method is employed to find suboptimal feedback control laws for the proof-mass actuators.

Proof-Mass Actuators

One of several designs for a proof-mass actuator is shown in Fig. 3. It incorporates two doughnut shaped samarium cobalt magnets and an annular soft iron yoke which combines the functions of moving mass and magnet. This assembly moves over a fixed coil whose current is controlled to produce the required force on the magnet, and thus the required reaction on the structure to be damped. Given this coil size, the maximum force available is limited by the current which can be carried by the coil without overheating. In the prototype design, it

was found possible to achieve a maximum force of 1.92 Newtons over a travel of plus or minus 0.0127 m (a total range of one inch). The mass of the moving yoke was 0.278 kg. The design provides for two sensors, an accelerometer which will be attached to the damper in a fully collocated application, and a sensor to measure the relative position of the proof-mass within the housing. The accelerometer is to be of the servo type, because it has to provide control signals down to zero frequency, while the position sensor is essential if the effects of striking against the stops which limit travel were to be avoided. In the design shown in Fig. 3, a linear variable differential transformer (LVDT) is used. However, an alternative sensor is also used quite successfully. This incorporated a tapered aluminum sleeve shrunk over the yoke (the yoke could have been tapered during manufacture) used in conjunction with a proximeter (which measures the distance to a conducting surface).

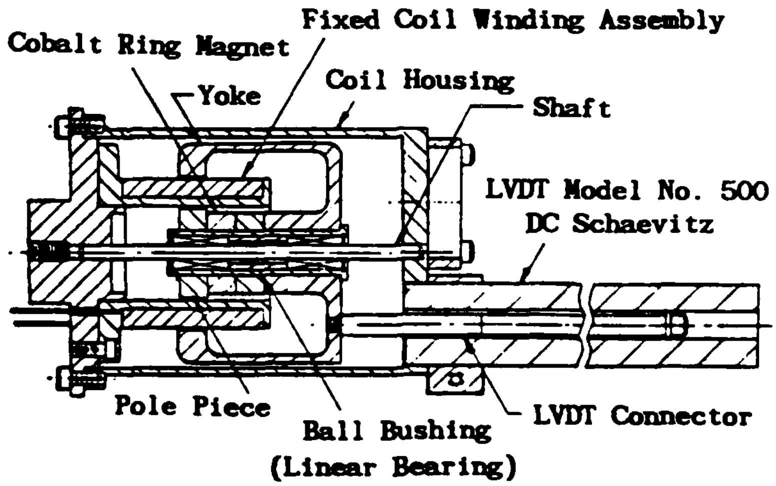

Fig. 3 A Schematic Diagram of a Proof-Mass Actuator

Problem Statement

The system to be considered, a single DOF mass-spring system, is shown in Fig. 4. A proof-mass actuator is attached at the end of the mass. The equations of motion of the system in Fig. 4 are

$$M\ddot{x}_1 + Kx_1 = u \qquad (70)$$
$$m\ddot{x}_2 = -u$$

The natural frequency of the spring-mass system is chosen to be

$$\omega = \sqrt{K/M} = 2\pi \text{ rad/s} = 1 \text{ Hz} \qquad (71)$$

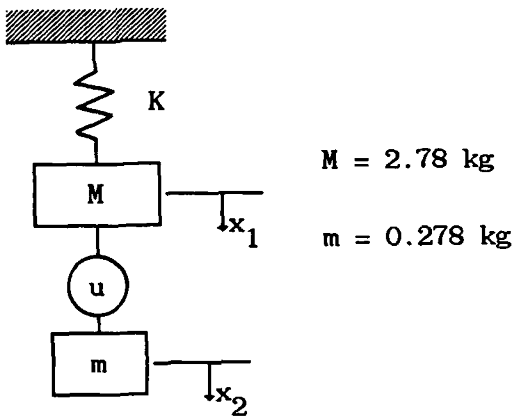

Fig. 4 A Single DOF Mass-spring System with the Proof-mass Actuator

Let $d = x_2 - x_1$ and introduce the state vector $\underline{s} = [x_1\ \dot{x}_1\ d\ \dot{d}]^T$. Then a set of first order differential equations is obtained.

$$\dot{\underline{s}} = A\underline{s} + Bu \qquad (72)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \omega^2 & 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1/M \\ 0 \\ -\dfrac{M+m}{Mm} \end{bmatrix} \tag{73}$$

Due to the physical limitations, the proof-mass actuator has constraints on the distance it can travel and the force that can be generated.

$$\begin{aligned} |\ d\ | &\le d_{max} = 1.27 \times 10^{-2} m \\ |\ u\ | &\le u_{max} = 2\ N \end{aligned} \tag{74}$$

The initial conditions are given by

$$\underline{s}(0) = [0.01m \quad 0 \quad 0 \quad 0]^T \tag{75}$$

The problem is described in two phases. The first phase is to find the optimal trajectories of control, $u^*(t)$, and the state variables, $\underline{s}^*(t)$, to bring the displacement of the mass M, x_1, within 2% of the initial value in the minimum settling time and the second phase is to find suboptimal feedback control laws based on the optimal trajectories using the methods for suboptimal design synthesis of feedback control systems.

Computation of LP/MST Trajectories

The computation can be done using one of the formulations given

before. The resulting LP/MST trajectories are shown in Fig. 5.

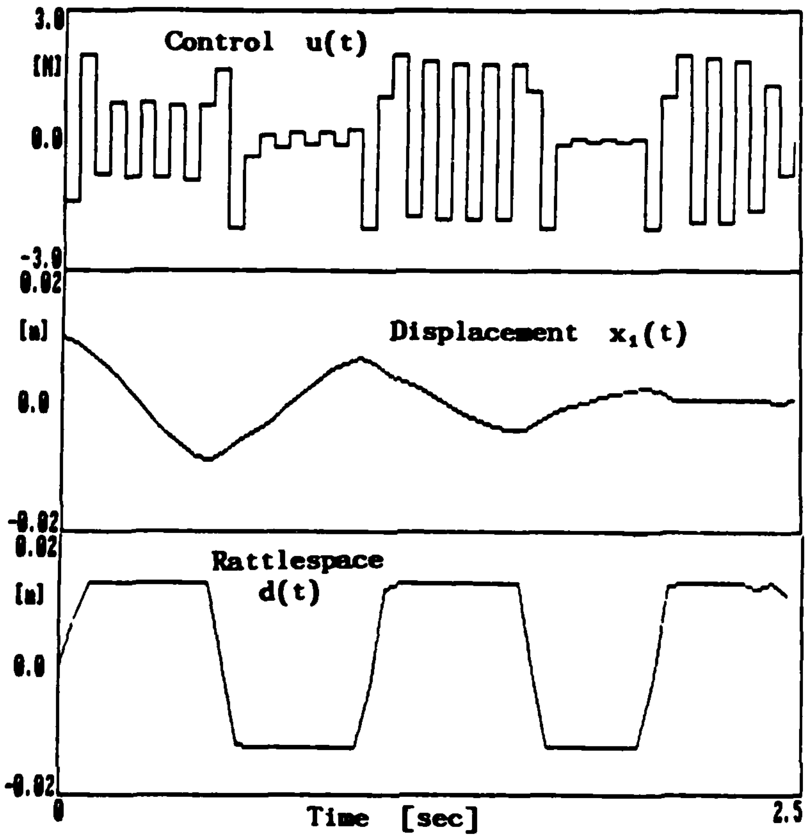

Fig. 5 LP/MST Trajectories

Identification of Feedback Control Based on Fitting Optimal Response Trajectories

The LP/MST trajectories shown in Fig. 5 are employed to find suboptimal feedback gain matrix D. In this subsection the method based on fitting optimal response trajectories is used. Since nu = 1 and 2n = 4, the problem becomes to find a vector $\underline{d}$ such as

$$u(k) = \underline{d}\ \underline{s}(k) \tag{76}$$

Using a curve fitting technique, the suboptimal feedback gains are obtained as

$$\underline{d} = [\ -81.86 \quad 11.50 \quad 30.26 \quad 8.017\ ] \tag{77}$$

and the resuting closed loop eigenvalues are

$$-107.4,\ -4.821,\ -0.5277 \pm 5.956i \tag{78}$$

The constraint on the control force is enforced by Eq. (48) with Eq. (74). The time response using these feedback gains for given initial conditions, Eq. (75), is shown in Fig. 6. Note that the constraint on the relative displacement is violated.

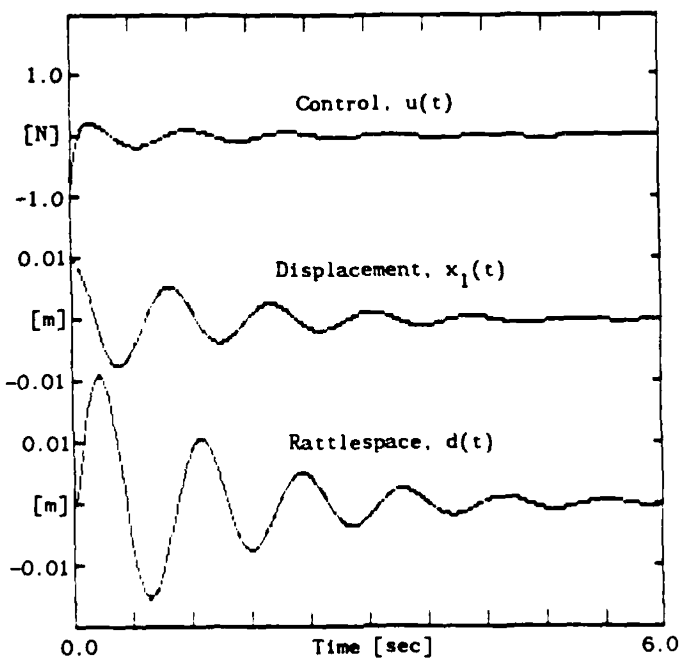

Fig. 6 Time Responses Using Suboptimal Feedback Gains

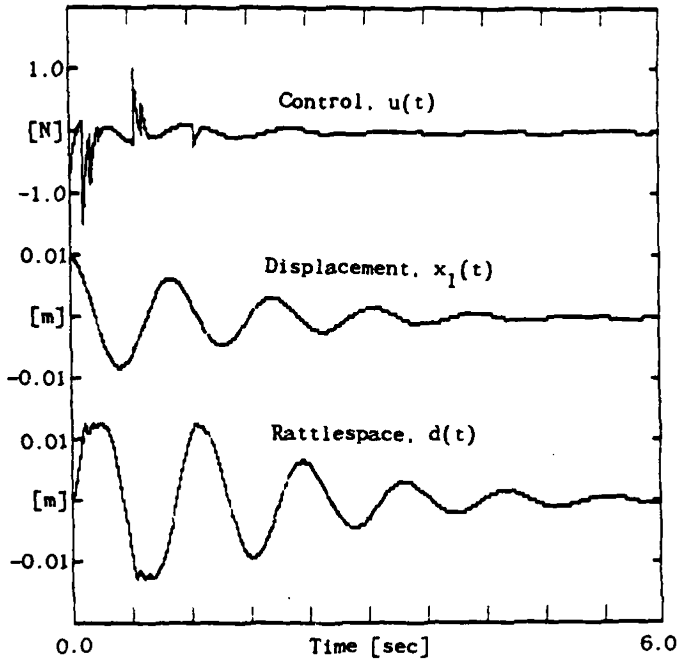

Fig. 7 Time Response Considering the Knocking

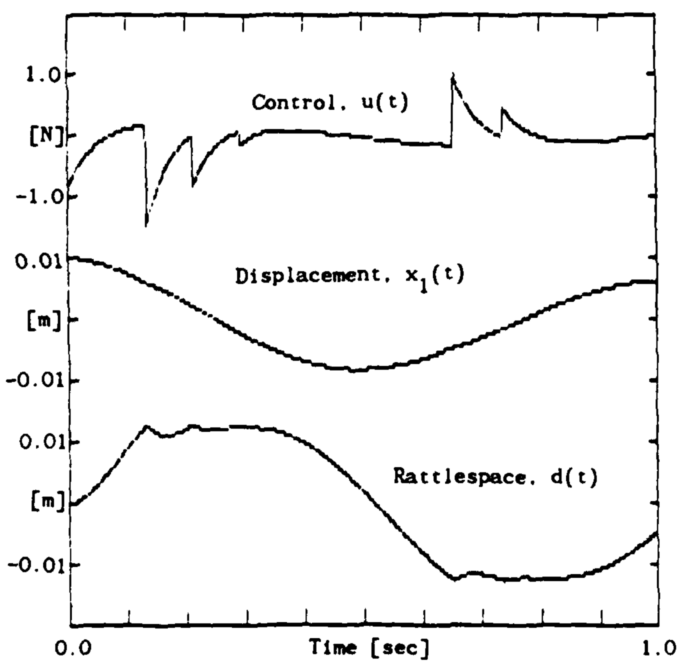

Fig. 8 A Closer Look on the Response During Knocking

To consider the influence of knocking between the proof-mass and the housing, time response is simulated by putting elastic impacts when the proof-mass violates the limit on the rattlespace and is shown in Figs. 7 and 8. Note that the knocking does not cause instability.

Identification of Feedback Control Based on the Assignment of Optimal Closed Loop Eigenvalues

The LP/MST trajectories shown in Fig. 5 are employed to find suboptimal feedback gain matrix D. In this subsection the method based on the assignment of optimal closed loop eigenvalues is used. Since the equations of motion are decoupled, the normal mode method is unnecessary for this problem. Optimal damping ratios and natural frequencies for the mass-spring system and the proof-mass are obtained based on the measure of approximation, Eq. (64). The resulting ζ^* and ω^* are $\zeta_1^* = 0.1087$ and $\omega_1^* = 6.3641$ for the mass-spring system and $\zeta_2^* = 0.0381$ and $\omega_2^* = 6.1479$ for the proof-mass actuator. Now, the closed loop eigenvalues to be assigned are determined from Eq. (66)

$$\begin{aligned} \lambda_{1,2} &= -0.6918 \pm 6.326i \\ \lambda_{3,4} &= -0.2342 \pm 6.143i \end{aligned} \tag{79}$$

Using an eigenvalue assignment technique, e.g., a method based on the generalized control canonical form, a feedback gain matrix D of Eq. (68) can be obtained as

$$D = [\ 9.827 \quad 0.3844 \quad 11.605 \quad 0.5517\ ] \tag{80}$$

and the resulting time responses are shown in Fig. 9. Note that the constraint on the rattlespace is not violated. However, more time is required to damp out the disturbance compared to the previous time responses of Fig. 7.

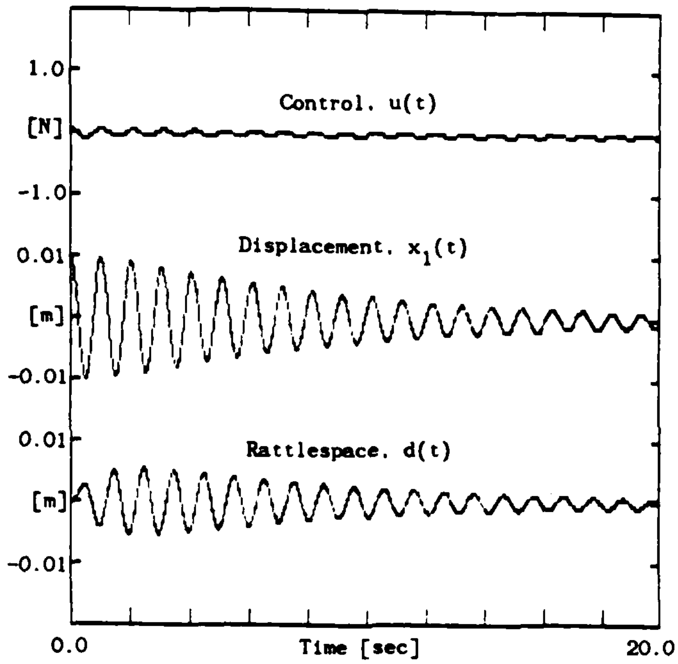

Fig. 9 Time Responses Using Suboptimal Feedback Gains

Position Loop Based Design Synthesis of Feedback Control

The two methods shown previously give systematic approaches for designing feedback control systems based on the LP/MST trajectories. Although the gains identified by fitting the optimal response trajectories give a better damped result, the constraints were violated

and stability would not be guaranteed for any control problems using this method. On the other hand, the gains obtained by assigning optimal closed loop eigenvalues give a stable control law without violating constraints, but more time is required to damp out the disturbance. To achieve stable, well-damped performance without violating constraints, the idea of a position loop is considered in conjunction with the identification method based on fitting optimal response trajectories.

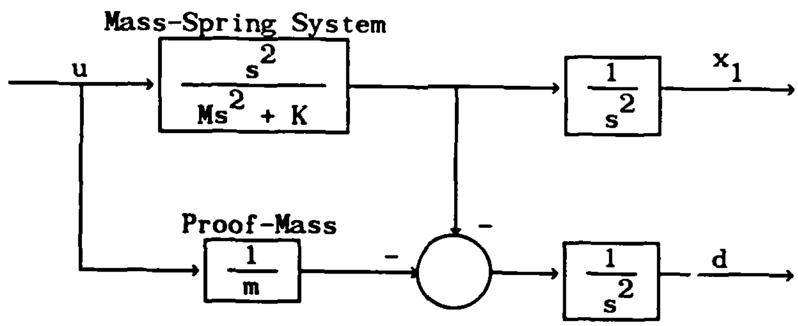

Fig. 10 Block Diagram of the System

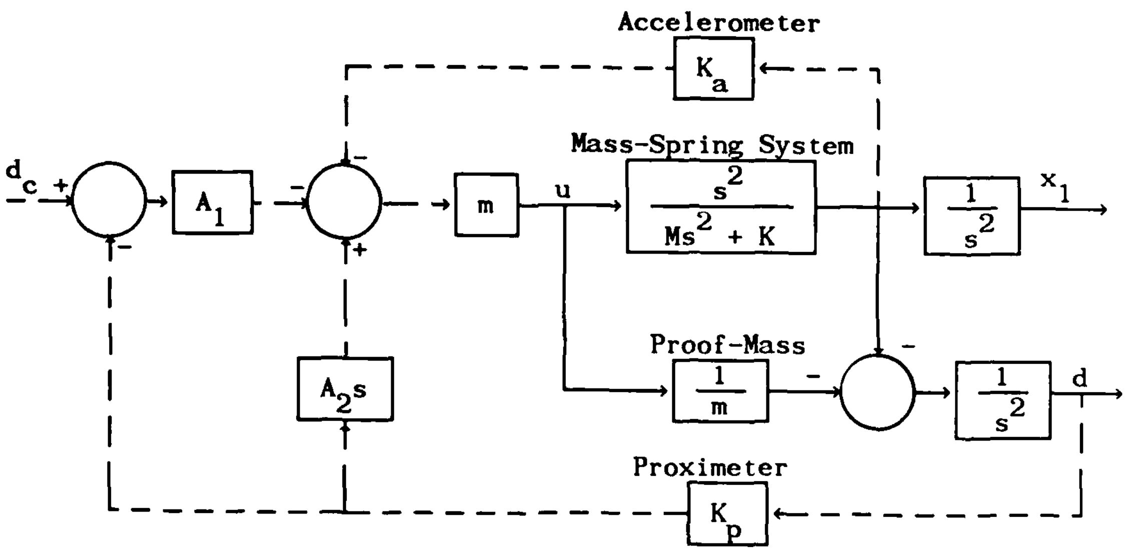

Fig. 11 Block Diagram with a Position Loop

The equations of motion in Eq. (70) can be represented by a block diagram. As implied in Fig. 5, the optimal motion of the proof-mass along its finite length track is rather simple compared to the trajectory of the control, u(t). Therefore, instead of utilizing the control force as an input to the system, the relative displacement of the proof-mass is used as the input control variable. The block diagram of Fig. 10 is expanded by attaching a position loop to the plant, as described by the dashed lines in Fig. 11. Now, the input to the system is the controlled relative position of the proof-mass d_c, while d represents the actual relative position. The accelerometer and proximeter feedback gains are K_a and K_p, while A_1 and A_2 are control gains. For $K_a = K_p = 1$ the equations of motion appear as

$$\ddot{x}_1 = (-Kx_1 + A_1md + A_2m\dot{d} - A_1md_c)/(M + m) \tag{81}$$

$$\ddot{d} = -A_1d - A_2\dot{d} + A_1d_c \tag{82}$$

and the resulting control force is given by

$$u = -(\ddot{d} + \ddot{x}_1)m \tag{83}$$

The state equation for the expanded system, using the state vector $\underline{s} = [\ x_1\ \dot{x}_1\ d\ \dot{d}\ ]^T$, becomes

$$\dot{\underline{s}}(t) = \tilde{A}\underline{s}(t) + \tilde{B}d_c \tag{84}$$

where

$$\tilde{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -K/(M+m) & 0 & A_1 m/(M+m) & A_2 m/(M+m) \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -A_1 & -A_2 \end{bmatrix} \quad \tilde{B} = \begin{bmatrix} 0 \\ -A_1 m/(M+m) \\ 0 \\ A_1 \end{bmatrix} \tag{85}$$

To determine the gains A_1 and A_2 in the position loop, consider a transfer function d/d_c

$$d/d_c = A_1/(s^2 + A_2 s + A_1) \tag{86}$$

where

$$A_1 = \omega_p^2 \quad \text{and} \quad A_2 = 2\zeta_p \omega_p \tag{87}$$

Select $\zeta_p = 0.707$ to minimize the settling time of the actual relative position in the position loop. The percentage overshoot of the system will be 4.325 %. To take the overshoot into consideration the maximum rattlespace should be reduced to

$$\bar{d}_{max} = 1.217 \times 10^{-2} m \tag{88}$$

Assume that the external excitation to the system will not exceed the control force level, i.e.,

$$\omega^2 |x_1|_{max} < \omega_p^2 \bar{d}_{max} = u_{max}/m \tag{89}$$

where ω_p is selected from Eq. (89)

$$\omega_p^2 = u_{max}/(m\bar{d}_{max}) \quad \text{or} \quad \omega_p = 24.3 \text{ rad/s} \tag{90}$$

Under these conditions the system will not violate the given constraints, Eq. (74).

The input d_c is determined using the identification method based on fitting optimal LP/MST trajectories, i.e., $d^*(k)$ and $\underline{s}^*(k)$ for k = 1,2,...,N. The task is to find gain $\underline{\tilde{d}}$ such that

$$d^*(k) \simeq \underline{\tilde{d}}\ \underline{s}^*(k) = d^s(k) \tag{91}$$

where

$$\underline{\tilde{d}} = [\ \tilde{d}_{x_1}\ \tilde{d}_{\dot{x}_1}\ \tilde{d}_d\ \tilde{d}_{\dot{d}}\ ] \tag{92}$$

Note that $\tilde{d}_d$ should be set equal to zero to obtain meaningful curve fitting results. Using a curve fitting technique, suboptimal feedback gains are obtained as

$$\underline{\tilde{d}} = [-0.4439 \quad -0.3878 \quad 0 \quad -0.0349] \tag{93}$$

and the resulting closed loop eigenvalues are

$$-31.40 \pm 26.89i \quad \text{and} \quad -2.092 \pm 2.595i \tag{94}$$

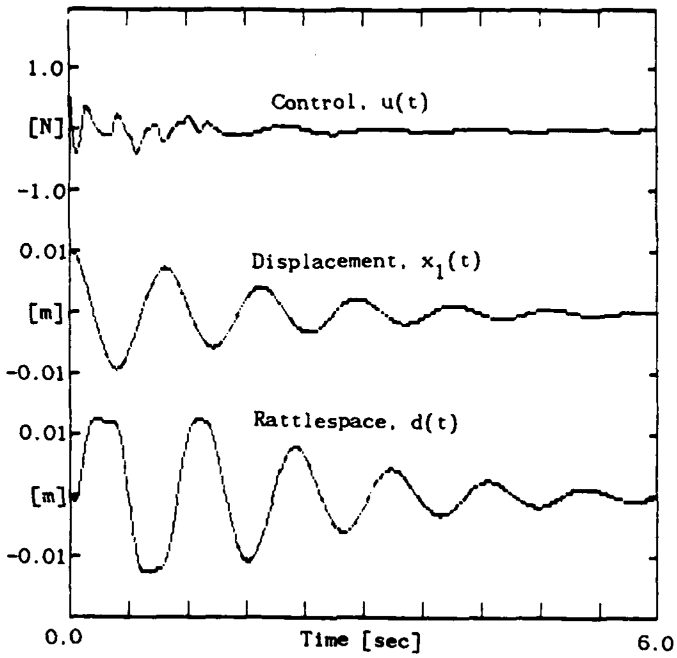

Fig. 12 Time Response Obtained by the Position Loop

The constraint on the rattlespace is enforced by

$$d_c(k) = \begin{cases} \bar{d}_{max} & \text{for} \quad d^s(k) > \bar{d}_{max} \\ d^s(k) & \text{for} \quad |d^s(k)| \leq \bar{d}_{max} \\ -\bar{d}_{max} & \text{for} \quad d^s(k) < -\bar{d}_{max} \end{cases} \tag{95}$$

For the initial conditions of Eq. (75), the time reponse is given in Fig. 12. Note that the figure shows a response similar to that of Fig. 7 without violating the rattlespace constraint.

IV. PUBLICATIONS

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2. Politansky, H. and Pilkey, W. D., "Suboptimal Feedback Vibration Control of a Beam with a Proof-Mass Actuator," AIAA Guidance, Navigation and Control Conference, Monterey, CA, August, 1987. (Submitted to the AIAA Journal of Guidance, Control, and Dynamics).

3. Haviland, J. K., Lim, T. W., Pilkey, W. D., and Politanski, H., "The Control of Linear Dampers for Large Space Structures," AIAA Guidance, Navigation and Control Conference, Monterey, CA, August, 1987. (Submitted to the AIAA Journal of Guidance, Control, and Dynamics).

4. Haviland, J. K., Politansky, H., Lim, T. W., and Pilkey, W. D., "The Control of Linear Proof-Mass Dampers," Sixth VPI & SU/AIAA Symposium on Dynamics and Control of Large Structures, June 29 - July 1, 1987.

5. Lim, T. W. and Pilkey, W. D., "Limiting Performance Characteristics of Transient Systems with l_1 Norm and Minimum Settling Time," Under Preparation.

6. Lim, T. W. and Pilkey, W. D., "Limiting Performance Based Feedback Control Synthesis for the Control of a Proof-Mass Actuator," Under Preparation.

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